Topology dependence of mass sensitivities in mode-localized sensors

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Abstract — Vibration mode localization has been employed as an ultrasensitive approach for mass detection and identification in recent years. Sensitivity enhancements of nearly three orders of magnitude relative to the more conventional resonant frequency shift approach have been experimentally demonstrated using this sensing paradigm, by either exploiting the critical dependence of the parametric sensitivity on the strength of internal coupling or by increasing the number of degrees of freedom by arraying multiple resonators with weak coupling springs. We propose here, for the first time, an additional approach to sensitivity enhancement in such *mode-localized* mass sensors by utilizing the sensitivity dependence on the operating frequency and the stiffness of the resonator topology. We experimentally demonstrate the sensitivity dependence on the topology, by comparing and contrasting the vibration behaviour of three pairs of electrically coupled microelectromechanical (MEM) resonators of different structural configurations and operating frequencies. The shifts in the eigenstates for the same relative mass additions are experimentally demonstrated to be over three orders of magnitude greater than corresponding resonant frequency shifts. They are also shown to increase proportionally with the square of the resonant frequencies of the coupled resonator platforms (with the stiffer configurations of higher operating frequencies yielding a further one order of magnitude improvement over the more compliant topologies of nearly equal resonator masses). This topology dependence while providing a systematic approach to the design of mode-localized mass sensors within a given design space, also suggests an alternate route to improving the mass sensitivity by designing stiffer topologies with weak coupling instead of the more conventional route of scaling down system dimensions in traditional resonant mass sensors.

INTRODUCTION

Over the past decade, micro- and nanofabricated resonant mass sensors have emerged as attractive platforms for the rapid and sensitive detection of biological and chemical analytes. Most of these sensors operate by measuring relative shifts in resonant frequency caused by molecule specific interactions of target analytes on the surface of the resonators [1, 2]. The naturally high frequency sensitivity due to the miniscule active masses of these resonant sensors and the quasi-digital nature of the output signal, have made this paradigm of mechanical sensing particularly attractive for analyte detection by mass. These attributes along with the continued trend towards miniaturizing system dimensions down to the sub-micron domain, have consequently translated into unprecedented opportunities for ultrasensitive resonant mass sensing, demonstrated by the recently achieved attogram [3] and zeptogram [4] resolutions.

In contrast, the concept of utilizing the phenomenon of vibration localization in arrays of weakly coupled, nearly identical resonators, has also been proposed as a highly sensitive mass sensing technique in recent years [5-7]. Sensitivity enhancements of nearly two to three orders of magnitude relative to the more conventionally used resonant frequency shift approach have been theoretically and experimentally demonstrated using this sensing paradigm, by either exploiting the critical dependence of the parametric sensitivity of such mode-localized mass sensors on the strength of internal coupling between the coupled resonators [5, 6] or by increasing the number of degrees of freedom by arraying multiple resonators with weak coupling springs [7]. In this paper, unlike the previous attempts in [5-7], we propose an additional route to sensitivity enhancement in such sensors by utilizing the sensitivity dependence on the operating frequency and the stiffness of the resonator topology.

THEORY

When mechanically identical resonators are coupled through weak springs, vibrations propagate evenly throughout the structure. An induced mass addition on one of the coupled structures, however, inhibits the propagation of vibration resulting in a confinement of vibration energy to small geometric regions. The extent of such vibration energy confinement or *localization* (for a given mass perturbation), depends not only on the magnitude of the periodicity breaking irregularity within the system, but also on the strength of the internal coupling spring constant between the resonators (with weaker coupling leading to stronger localization) and the operating frequency of the coupled resonator platforms. Measuring the shifts in the eigenstates due to such induced mass additions may yield sensitivities as high as orders of magnitude greater than corresponding resonant frequency variations under conditions of weak internal coupling [6]. In order to understand the underlying physics behind the phenomenon, consider a pair of resonators coupled through a weak coupling spring (k_c) as represented in the discretized model shown in Fig. 1. When the two resonators are perfectly identical ($m_1 = m_2 = m$; $k_1 = k_2 = k$) the system is symmetric about k_c.

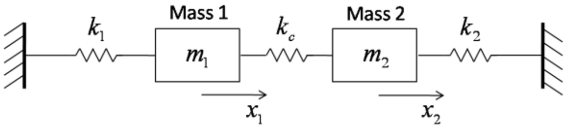

Fig. 1: Lumped element model of a coupled 2 degree-of-freedom spring mass system.

An addition of mass (Δm) on either of the coupled structures, however, breaks the symmetry causing a modification in both the eigenvalues and the normalized mode shapes (eigenstates). So long as the relative change in mass is smaller than the scaled coupling factor $\left(\Delta m/m \ll k_c/k\right)$, the relative shifts in the eigenstates may be derived using the perturbation approach to be [6],

$$\left|\frac{\Delta u_i}{u_{0i}}\right| \approx \frac{\Delta m}{m}\left(\frac{k}{4k_c}\right);(i=1,2) \tag{1}$$

Comparing this with the conventional approach of measuring relative shift in the resonant frequency (refer (2)), it may be observed that the relative variations in the eigenstates are dependent not only on the magnitude of relative mass shift as in the case of resonant frequency variations $(\delta = \Delta m/m)$, but also on the strength of internal coupling (k_c) and the stiffness of the coupled resonator platform (k). For any value of $k_c < (k/2)$, the eigenstate shifts can be made greater than those in resonant frequency –

$$\left|\frac{\Delta f}{f_0}\right| \approx \frac{\Delta m}{2m} \tag{2}$$

This critical dependence of the parametric sensitivity on the strength of internal coupling (k_c) has been exploited in recent years to realize mode-localized mass sensors with sensitivities that are as high as two to three orders of magnitude greater than those in frequency [5, 6]. Further sensitivity enhancements have also been demonstrated by increasing the number of degrees of freedom within the system by arraying multiple resonators with weak coupling springs [7]. In an attempt to improve the sensitivity of mode-localized mass sensors even further, we propose here, an additional approach to sensitivity enhancement in this sensing paradigm by utilizing the critical dependence of mass sensitivity on the stiffness of resonator topology. The concept is experimentally demonstrated by comparing and contrasting vibration behaviours of three pairs of electrically coupled, nearly identical resonators of different structural configurations and operating frequencies but with nearly equal effective dynamic masses. The shifts in eigenstates while being over three orders of magnitude greater than those in resonant frequency for the same relative mass additions, are also experimentally demonstrated to increase proportionally with the stiffness of the resonator topologies (as derived in (1)). Such a topology dependence of mass sensitivity in mode-localized sensors suggests an alternate/additional route to enhancing the parametric sensitivity of such sensors by designing stiffer topologies with weak coupling instead of the more conventional route of scaling down system dimensions in traditional resonant mass sensors.

EXPERIMENT

The topologies considered here include pairs of electrically coupled, nearly identical microelectromechanical flexural wine glass mode ring resonators, double free-free beams (DFFs) and double ended tuning forks (DETFs). The devices were fabricated in a commercial foundry process using the silicon-on-insulator microelectromechanical systems (SOI-MEMS) process through MEMSCAP Inc., USA. The fabricated devices were tested under partial vacuum (of approx. 10mTorr) in a custom vacuum chamber. Each of the resonators in all three configurations was driven and sensed using capacitive transduction. The resonator pairs in all three configurations were separated by a designed coupling gap of about 2 μm. DC voltages of equal magnitudes but opposite polarities were applied on each of resonators to establish a *voltage-tunable* electrical coupling spring between the closely spaced resonator pairs, without disturbing the initial symmetry of the nearly periodic system [6]. After measuring the initial vibration behaviour of each of the devices (deduced from the relative transmission responses of each of the coupled resonators measured from the S21 (scattering) parameter-frequency response observed on a Network analyser), platinum patches of known dimensions (and correspondingly of known masses) were deposited on one of coupled resonators of each configuration to measure the relative shifts in the eigenstates and eigenvalues. The shifts in the eigenstates were then compared and contrasted, first with those of the eigenvalues and next, between the resonator topologies to establish the topology dependence of mass sensitivities in this sensing paradigm.

Topology 1: Flexural wine glass mode rings

An optical micrograph of the coupled flexural wine glass mode ring resonators is shown in Fig. 3(a). The dimensions of the device are elaborated in Table 1.

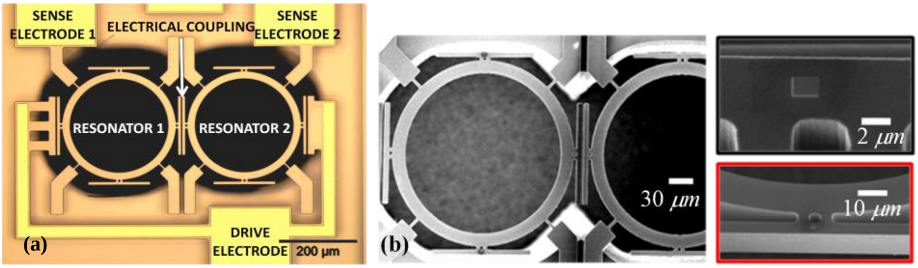

Fig. 3. Optical micrograph and scanning electron microscope (SEM) images of the electrically coupled ring resonators (a) before and (b) after mass deposition using FIB.

Table 1: Electrically coupled flexural wine glass mode ring resonator parameters and dimensions.

Quantity	Value	Unit
Device thickness	10	μm
Inner diameter	190	μm
Outer diameter	220	μm
Attached plate length	120	μm
Attached plate width	6	μm
Effective mass (approx.)	180	ng
Effective stiffness (approx.)	11200	N/m
Mass of platinum deposited on one of the rings (approx.)	12.7	pg
Relative mass perturbation ($\delta = \Delta m/m$) (approx.)	$7{*}10^{-5}$	-

Actuation was achieved using parallel plates of equal nominal dimensions attached to the anti-nodal points of each of the rings (Fig. 3(a)). An electrical coupling spring of magnitude ≈ 2 N/m was then established between the rings by subjecting them to equal but opposite DC polarization voltages of magnitude +15V and -15V. After measuring the initial eigenstates, two platinum patches (measuring 1.848 μm x 1.909 μm x 84 nm and 1.882 μm x 1.856 μm x 87 nm approx., corresponding to a mass of approx. 12.7pg), were deposited on two diametrically opposite anti-nodal parallel plates of one of the rings (resonator 1) using the FIB (Fig. 3(b)).

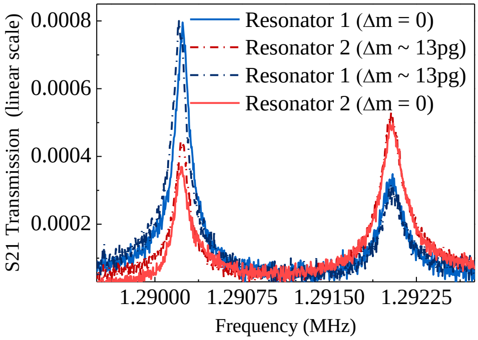

Fig. 4. Experimentally measured relative transmission responses of resonators 1 and 2 before and after mass deposition on resonator 1.

The variations in the eigenstates were then studied and compared with those in frequency. The relative transmission responses of resonators 1 and 2 before and after mass addition are shown in Fig. 4. It can be noticed that the addition of mass on resonator 1 resulted in a relative shift in the eigenstates of about 4.32% and 3.448% at the two eigenvalues, while the relative variation in the resonant frequency corresponded to about 0.00237% indicating an improvement in parametric sensitivity for the same deposited mass of over three orders of magnitude.

Topology 2: Double free-free beam (DFF) resonators

A similar test was then carried out on a pair of electrically coupled DFF resonators, an optical micrograph of which is shown in Fig. 5(a). The dimensions of each of the coupled free-free beams are elaborated in Table 2. After deducing the eigenstates and eigenvalues of the system for an induced electrical coupling spring of magnitude ≈ 4.42 N/m between the resonators, the relative shifts in the eigenstates and those in the eigenvalues were measured experimentally for an induced mass deposition of 17pg (approx.) on each beam of resonator 1 (Fig. 5(b)). The relative transmission responses of resonators 1 and 2 before and after mass addition are shown in Fig. 6.

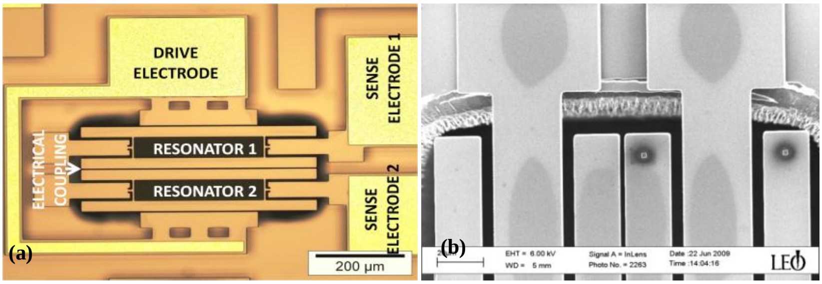

Fig. 5. Optical micrograph and SEM images of double free-free beam (DFF) resonators (a) before mass addition and (b) after mass deposition using the FIB.

Table 2: Electrically coupled double free-free (DFF) beam parameters and dimensions.

Quantity	Value	Unit
Device thickness	25	μm
Beam length	400	μm
Beam width	20	μm
Effective mass (approx.)	110	ng
Effective stiffness (approx.)	3020	N/m
Mass of platinum deposited on each of the beams (approx.)	17	pg
Relative mass perturbation ($\delta = \Delta m/m$) (approx.)	$1.46*10^{-4}$	-

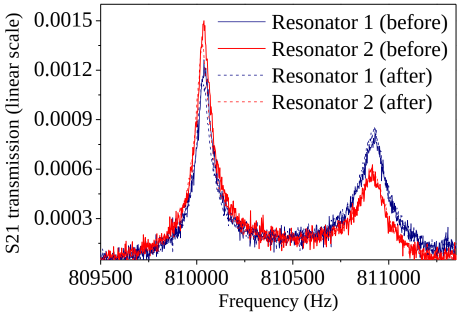

Fig. 6. Experimentally measured relative transmission responses of resonators 1 and 2 before and after mass deposition on resonator 1.

It may be noticed from Fig. 6 that the addition of mass on resonator 1 resulted in a relative shift in the eigenstates of about 1.42%, while the relative variation in the resonant frequency corresponded to about 0.0049%. As expected from (2), a reduction in the dynamic mass of the resonator (relative to that of the rings) resulted in a larger relative shift in resonant frequency for nearly the same magnitude of mass addition. The eigenstate variations however, reduced to 1.42% (in comparison to that of the rings (4.3%)) indicating a dependence of the eigenstate sensitivity on the stiffness of the resonator and the coupling spring as predicted by (1).

Topology 3: Double ended tuning fork (DETF) resonators

The next topology considered corresponded to a pair of nearly identical DETF resonators of effective dynamic mass similar to that of the rings but of operating stiffness nearly one order of magnitude lesser than the ring resonators (Fig. 7 (a)). The dimensions of the tuning forks are elaborated in Table 3. In the presence of an electrical coupling spring of magnitude approx. 1 N/m, an induced mass deposition of approximately 12.5 pg on each beam of one of the tuning forks (refer Fig. 7(b)) using the FIB, resulted in a variation in the eigenstates of nearly 1%, while that in resonant frequency corresponded to 0.00213% (refer Fig. 8). As expected from (2), the measured shifts in resonant frequency were very similar to that of the rings (as the magnitude of relative mass perturbation are nearly equal in both cases). However, measured eigenstate variations were much lower than the rings due to the lower dynamic stiffness offered by the tuning forks relative to that of the rings (refer Table 3, Fig. 8 and Fig. 9).

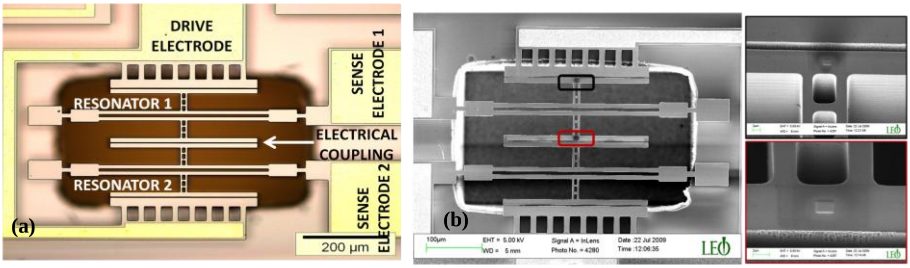

Fig. 7. Micrograph and SEM images of DETFs (a) before and (b) after mass deposition.

Table 3: Electrically coupled DETF resonator parameters and dimensions.

Quantity	Value	Unit
Device thickness	25	μm
Beam length	300	μm
Beam width (approx.)	7	μm
Effective mass (approx.)	182	ng
Effective stiffness (approx.)	830	N/m
Mass of platinum deposited on each of the beams (approx.)	12.5	pg
Relative mass perturbation ($\delta = \Delta m/m$) (approx.)	$6.9*10^{-5}$	-

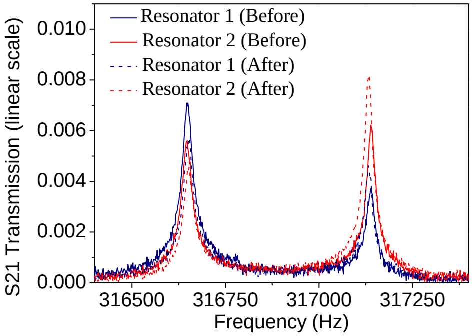

Fig. 8. Experimentally measured relative transmission responses of resonators 1 and 2 before and after mass deposition on resonator 1.

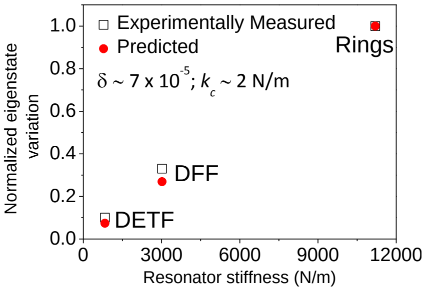

Fig. 9. Normalized eigenstate variations as a function of stiffness of resonator topology.

Comparing the eigenstate variations from all three topologies to those of the rings and plotting the resulting shifts as a function of stiffness yielded Fig. 9. Observe that for a fixed relative mass addition and coupling spring constant, the sensitivity scales proportionally as a function of the stiffness of the resonator topology considered with stiffer structures at higher operating frequencies (rings) yielding a further one order of magnitude improvement in parametric sensitivity over the more compliant topologies (DETF). This scaling behaviour is consistent with predictions from theory.

CONCLUSIONS

This paper demonstrates the sensitivity dependence of mode-localized mass sensors on the operating frequency and stiffness of the resonator topologies. Relative shifts in the eigenstates that are over three orders of magnitude greater than corresponding resonant frequency variations for the same induced mass perturbations are experimentally demonstrated. The eigenstate shifts are also shown to scale proportionally as a function of the stiffness of the resonator topology with stiffer configurations of higher operating frequency yielding a further one order of magnitude improvement in parametric sensitivity over the more compliant topologies.

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